The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 3, 2024 Course: EE 313 Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

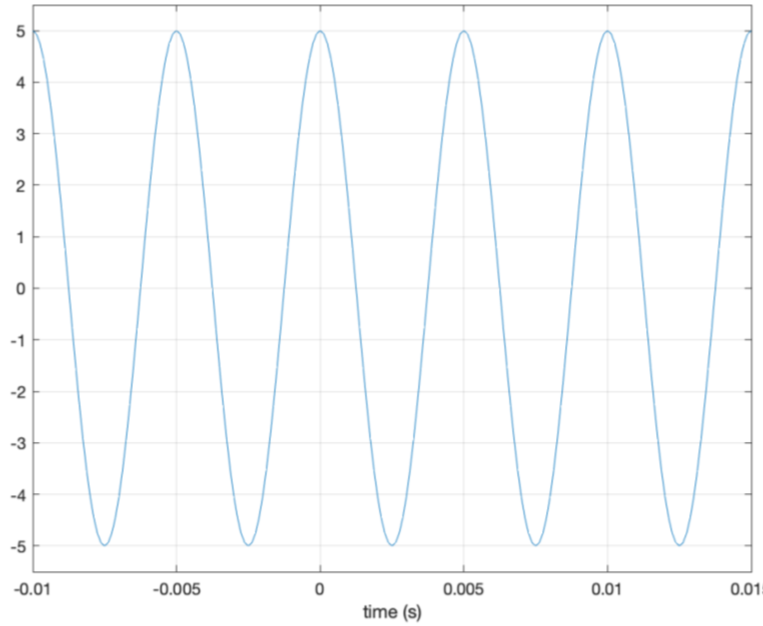
Problem	*Point Value*	*Your score*	*Topic*
1	24		Sinusoidal Signals
2	24		Fourier Series
3	26		Sampling
4	24		Potpourri
Total	100		

Problem 1.1 *Sinusoidal Signals*. 24 points.

Consider the sinusoidal signal
$x(t) = A\cos(2\,\pi\, f_0\, t\; +\; \theta)$ with

- amplitude A
- continuous-time frequency f_0 in Hz
- phase θ in radians

From the plot of $x(t)$ on the right,



(a) Estimate the amplitude A. Explain how you estimated the value of this parameter. *6 points.*

(b) Estimate the continuous-time frequency f_0 in Hz. Explain how you estimated the value of this parameter. *6 points.*

(c) Estimate the phase θ in radians. Explain how you estimated the value of this parameter. *6 points*.

(d) What is the phase of the signal $x(t - 0.001)$? Please show your intermediate steps. *6 points*.

Problem 1.2. *Fourier Series Properties*. 24 points.

The continuous-time Fourier series has several properties.

For example, if $y(t) = A\, x(t)$ and $x(t)$ is periodic with fundamental frequency f_0 and Fourier series coefficients a_k, then the Fourier series coefficients b_k for $y(t)$ can be found using $b_k = A\, a_k$:

$$y(t) = A\, x(t) = A \sum_{k=-\infty}^{\infty} a_k\, e^{j2\pi(k f_0)t} = \sum_{k=-\infty}^{\infty} A\, a_k\, e^{j2\pi(k f_0)t}$$

For the following expressions, derive the relationship between the Fourier series coefficients b_k for $y(t)$ and the Fourier series coefficients a_k for $x(t)$ where

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

(a) $y(t) = x(t) + C$ where C is a constant. *6 points*.

(b) $y(t) = \sin(2\,\pi\, f_0\, t)\, x(t)$. This is a type of amplitude modulation. *9 points*.

Hint: If $y(t) = \cos(2\,\pi\, f_0\, t)\, x(t)$, then $b_k = \frac{1}{2}\, a_{k-1} + \frac{1}{2}\, a_{k+1}$.

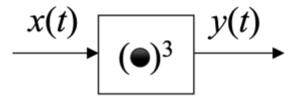
(c) $y(t) = \frac{d}{dt}\, x(t)$. *9 points*.

Problem 1.3. *Sampling.* 26 points**.**

(a) Let $x(t) = \cos(2\pi f_0 t)$ be a continuous-time signal for $-\infty < t < \infty$.

i. From the block diagram on the right, $y(t) = x^3(t)$.

Write $y(t)$ as a sum of cosines. *6 points*.

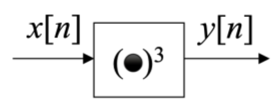


ii. Let $f_0 = 3000\ Hz$. What negative, zero, and positive frequencies are present in $y(t)$? *6 points*

(b) Let $x(t) = \cos(2\pi f_0 t)$ be a continuous-time signal for $-\infty < t < \infty$. Discrete-time signal $x[n]$ is obtained by sampling $x(t)$, and $y[n]$ is obtained by sampling $y(t)$.

i. From the block diagram below, $y[n] = x^3[n]$.

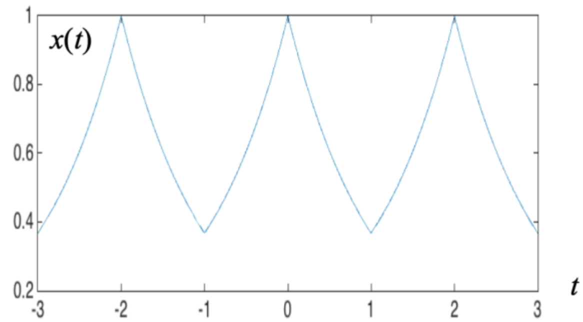
Write it as a sum of cosines. *6 points*



ii. Let $f_0 = 3000$ Hz and $f_s = 8000$ Hz. What negative, zero and positive discrete-time frequencies are present in $y[n]$ between –π rad/sample and π rad/sample? What are their corresponding continuous-time frequencies? *8 points*.

Problem 1.4. *Potpourri*. 24 points.

(a) Consider the periodic signal $x(t)$ on the right with a fundamental period of $T_0 = 2$ seconds.



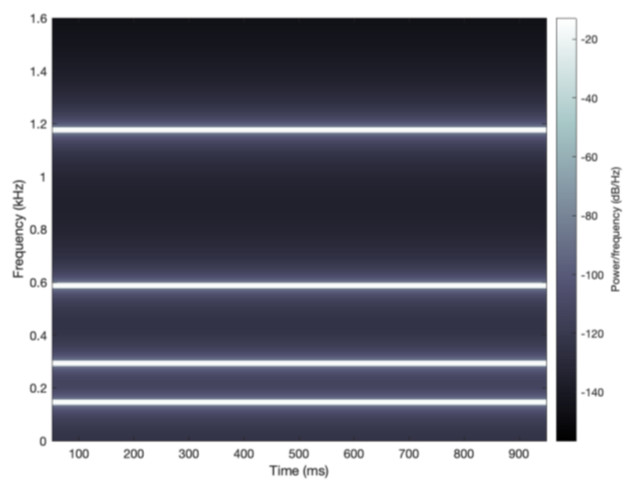
Over one fundamental period,

$$e^t \quad \text{for } -\frac{T_0}{2} \le t < 0$$

$$e^{-t} \quad \text{for } 0 \le t < \frac{T_0}{2}$$

If we keep a large but finite number of Fourier series coefficients, explain whether or not the Fourier synthesis will suffer from Gibbs phenomenon. In your answer, please explain what Gibbs phenomenon is. *12 points*.

(b) Consider the spectrogram of a signal $y(t)$ given below. *12 points*.



i. How would you describe the relationship among the frequencies in $y(t)$.

ii. Please give an equation for $y(t)$ over the time plotted $0 \le t \le 1$.